ARJUNA

JEE 2026 BATCH

MATHEMATICS

BINOMIAL THEOREM

Lecture No- 09

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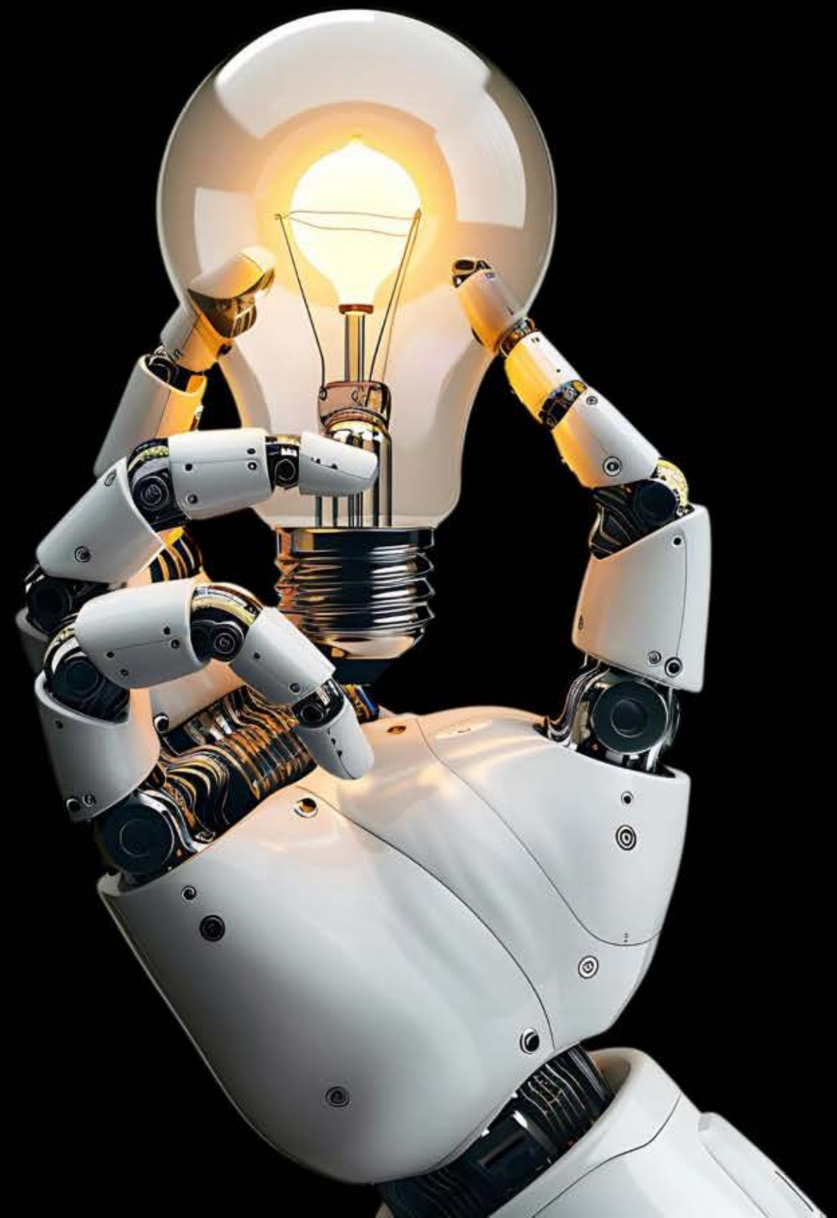
Topics *to be covered*

1. Double Sigma Problems
2. Multinomial Theorem
3. Question Practice
4.



LAST CLASS RECAP

Bino-Binomial Series

$${}^{23}C_1\,{}^{24}C_{24} \pm {}^{23}C_2\,{}^{24}C_{23} + {}^{23}C_3\,{}^{24}C_{22} \pm \ldots\ldots + {}^{23}C_{23}\,{}^{24}C_2 = {}^{47}C_{25}.$$

coeff of x^{25} in $(1+x)^{23}\cdot(1+x)^{24}$

coeff of x^{25} in $(1+x)^{23}\cdot(1-x)^{24}$ =

$${}^{23}C_1\cdot{}^{24}C_{24}(-1)^{24} + {}^{23}C_2\,{}^{24}C_{23}(-1)^{23} + {}^{23}C_3\,{}^{24}C_{22}(-1)^{22} + \ldots$$

$$(1+x)^{23}\ (1-x)^{24}$$

$$(1+x)^{23}\cdot(1-x)^{23}\cdot(1-x)^{1}$$

$$(1-x^2)^{23}\cdot(1-x)$$

$${}^{23}C_r(-x^2)^r$$

$${}^{23}C_r(-1)^r\cdot x^{2r}$$

$r = 12$

$$(0)x^{25}\times(1)x^0 = 0\ x^{25}$$

$$({}^{23}C_{12})x^{24}\times(-1)x^1 = -{}^{23}C_{12}\ x^{25}$$

fnf.

$$(1+7x)^{23} \cdot (1+x)^{23} \cdot (1+x)^{1}$$

$$(1+8x+7x^2)^{23} \cdot (1+x)^{1}$$

M.N.

$$(\quad)x^{25} \times (\quad)x^{0}$$

$$(\quad)x^{24} \times (\quad)x^{1}$$

QUESTION [IIT-JEE-2010]

$(1+x)^{10} \longrightarrow {}^{10}C_r\, x^r$ → A_r.

For $r = 0, 1, \ldots, 10$, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}, (1 + x)^{20}$ and $(1 + x)^{30}$.

Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

$A_r = {}^{10}C_r$, $B_r = {}^{20}C_r$, $C_r = {}^{30}C_r$.

$\sum_{r=1}^{10} {}^{10}C_r \left({}^{20}C_{10} \cdot {}^{20}C_r - {}^{30}C_{10}\, {}^{10}C_r \right)$

$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_r\, {}^{20}C_r - {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_r\, {}^{10}C_r$.

${}^{30}C_{10} - {}^{20}C_{10}$.

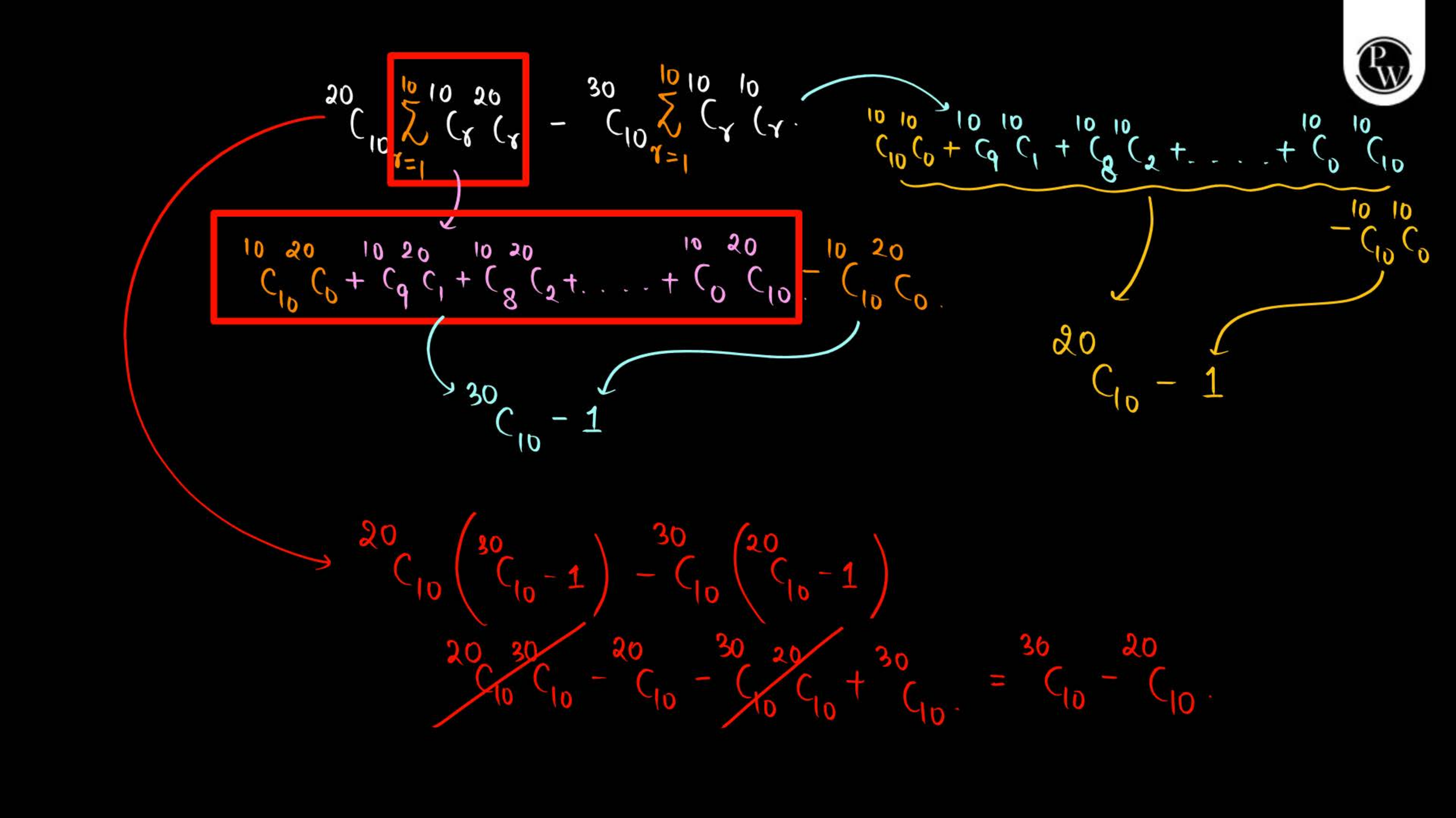
$C_{10} - B_{10}$.

(A) $B_{10} - C_{10}$

(B) $A_{10}(B_{10}^2 - C_{10}A_{10})$

(C) 0

(D) $C_{10} - B_{10}$


$^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_r\,{}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_r\,{}^{10}C_r$
$^{10}C_{10}\,{}^{20}C_0 + {}^{10}C_9\,{}^{20}C_1 + {}^{10}C_8\,{}^{20}C_2 + \ldots + {}^{10}C_0\,{}^{20}C_{10} - {}^{10}C_{10}\,{}^{20}C_0$
$^{10}C_{10}\,{}^{10}C_0 + {}^{10}C_9\,{}^{10}C_1 + {}^{10}C_8\,{}^{10}C_2 + \ldots + {}^{10}C_0\,{}^{10}C_{10}$
$-{}^{10}C_{10}\,{}^{10}C_0$
$^{30}C_{10} - 1$
$^{20}C_{10} - 1$
$^{20}C_{10}\left({}^{30}C_{10} - 1\right) - {}^{30}C_{10}\left({}^{20}C_{10} - 1\right)$
$^{20}C_{10}\,{}^{30}C_{10} - {}^{20}C_{10} - {}^{30}C_{10}\,{}^{20}C_{10} + {}^{30}C_{10} = {}^{30}C_{10} - {}^{20}C_{10}$

CHALLENGER QUESTION

If $\sum_{r=0}^{1000} \frac{\binom{1000}{r}}{\binom{2019}{r}} = \frac{m}{n}$, ($m$ and n are relatively prime positive integers) then $m+n =$ ___ 152

$$\frac{1}{{}^{2019}C_{1000}} \sum_{r=0}^{1000} {}^{2019-r}C_{1019}$$

$$\frac{1000!\, r!\, (2019-r)! \times 1019!}{r!\,(1000-r)! \times 2019! \times (1019)!}$$

$${}^{2019-r}C_{1019}$$

$$\frac{}{{}^{2019}C_{1000}}$$

$${}^{2020}C_{1020} = {}^{2019}C_{1019} + {}^{2018}C_{1019} + {}^{2017}C_{1019} + \ldots + {}^{1019}C_{1019}$$

$$\frac{}{{}^{2019}C_{1000}}$$

$$\frac{2020!\, 1000!\, 1019!}{1020! \times 1000! \times 2019!} = \frac{2020}{1020} = \frac{202}{102} = \frac{101}{51}$$

Ques:

$$\left(1+x+x^2\right)^7 = \sum_{r=0}^{14} a_r \cdot x^r$$

$$\left(1+x+x^2\right)^7 = a_0 + a_1x^1 + a_2x^2 + \dots\dots + a_{14}x^{14}$$

$x=1$

$$3^7 = a_0 + a_1 + a_2 + \dots\dots + a_{14}$$

$$1 = a_0 - a_1 + a_2 - a_3 + \dots + a_{14}$$

$$\frac{3^7+1}{2} = \left(a_0 + a_2 + a_4 + \dots + a_{14}\right)$$

$$\frac{3^7-1}{2} = \left(a_1 + a_3 + a_5 + \dots + a_{13}\right)$$

Ques:

$$(1+x+x^2)^7 = \sum_{r=0}^{14} a_r \cdot x^r$$

$$(1+x+x^2)^7 = a_0 + a_1x^1 + a_2x^2 + a_3x^3 - - - - + a_{14}x^{14}$$

diff.

$$7(1+x+x^2)^6 \cdot (1+2x) = 1a_1 + 2a_2x + 3a_3x^2 + - - - - - + 14\,a_{14}x^{13}$$

$x=1$

$$7 \cdot (3)^7 = 1a_1 + 2a_2 + 3a_3 + - - - - + 14 \cdot a_{14}$$

$$?? = 1^2a_1 + 2^2a_2 + 3^2a_3 + - - - - + 14^2 \cdot a_{14}$$

DIBY (A)

Ques:

$$(1+x+x^2)^7 = \sum_{r=0}^{14} a_r \cdot x^r$$

$a_5 = ??$ → coeff. of $x^5 = a_5$.

Bino.

$$(1+x+x^2)^7 = a_0 + a_1x^1 + a_2x^2 + a_3x^3 - - - - + a_{14}x^{14}.$$

$$\left(1 + (x+x^2)\right)^7 = {}^7C_0 + {}^7C_1(x+x^2)^1 + {}^7C_2(x+x^2)^2 + {}^7C_3(x+x^2)^3 + {}^7C_4(x+x^2)^4$$

$$+ {}^7C_5(x+x^2)^5 + {}^7C_6(x+x^2)^6 + {}^7C_7(x+x^2)^7.$$

$(x+x^2)^3$ → ${}^3C_r\, x^{3+r}$; $(x+x^2)^4$ → ${}^4C_r\, x^{4+r}$; $(x+x^2)^5$ → ${}^5C_r\, x^{5+r}$

$(x+x^2)^n$ →

coeff of $x^5 = {}^nC_r\, x^{n-r} \cdot x^{2r}$

$= {}^nC_r\, x^{n+r}$.

$$a_5 = \left({}^7C_3\,{}^3C_2 + {}^7C_4\,{}^4C_1 + {}^7C_5\,{}^5C_0\right) \text{ Ans.}$$

Ques:

$$\left(1+x+x^2\right)^7 = \sum_{r=0}^{14} a_r \cdot x^r$$

$$a_5 = \left({}^7C_3\,{}^3C_2 + {}^7C_4\,{}^4C_1 + {}^7C_5\,{}^5C_0\right) \text{ fnf}$$

$$= \frac{7!}{4!2!} + \frac{7!}{3!3!} + \frac{7!}{2!5!}$$

$$\left(1+x+x^2\right)\left(1+x+x^2\right)\left(1+x+x^2\right)\left(1+x+x^2\right)\left(1+x+x^2\right)\left(1+x+x^2\right)\left(1+x+x^2\right)$$

$$x^1 \quad x^1 \quad x^2 \quad 1 \quad 1 \quad x^1 \quad 1 = 1 \cdot x^5$$

$$x^1 \quad x^1 \quad x^1 \quad x^1 \quad x^0 \quad x^0 \quad x^0 = 1 \cdot x^5$$

#	2, 2, 1, 0, 0, 0, 0	OR	$x^2\ x^2\ x^1\ x^0\ x^0\ x^0\ x^0$
#	2, 1, 1, 1, 0, 0, 0	OR	$x^2\ x^1\ x^1\ x^1\ x^0\ x^0\ x^0$
#	1, 1, 1, 1, 1, 0, 0	OR	$x^1\ x^1\ x^1\ x^1\ x^1\ x^0\ x^0$

$$\frac{7!}{2!4!} + \frac{7!}{3!3!} + \frac{7!}{2!5!}$$

Ques:

$$\left(1+x+x^2\right)^7 = \sum_{r=0}^{14} a_r \cdot x^r$$

(#) $a_{12} = ??$

$$\left(1+x+x^2\right)^7 = a_0 + a_1x^1 + a_2x^2 + a_3x^3 - - - - + a_{14}x^{14}$$

$x \to \frac{1}{x}$

$$\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^7 = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \frac{a_3}{x^3} + \ldots\ldots + \frac{a_{13}}{x^{13}} + \frac{a_{14}}{x^{14}}$$

$$\frac{\left(x^2+x+1\right)^7}{x^{14}} = \left(\frac{x^2+x+1}{x^2}\right)^7 = \frac{a_0x^{14} + a_1x^{13} + a_2x^{12} + a_3x^{11} + \ldots\ldots + a_{13}x^1 + a_{14}}{x^{14}}$$

$$\left(x^2+x+1\right)^7 = a_0x^{14} + a_1x^{13} + a_2x^{12} + a_3x^{11} + \ldots\ldots + a_{13}x^1 + a_{14}$$

Ques:

$$\left(1+x+x^2\right)^{\overset{n}{\downarrow}\,7} = \sum_{r=0}^{14} a_r \cdot x^r$$

(#) $a_{12} = a_2$

(#) $a_0 = a_{14}$

(#) $a_{13} = a_1$

(#) $a_5 = a_9$

$$\left(1+x+x^2\right)^7 = \boxed{a_0} + a_1x^1 + a_2x^2 + a_3x^3 - - + a_{12}x^{12} + a_{13}x^{13} + a_{14}x^{14}$$

$$\left(x^2+x+1\right)^7 = a_0x^{14} + a_1x^{13} + a_2x^{12} + a_3x^{11} + \ldots\ldots + a_{13}x^1 + \boxed{a_{14}}$$

$$\boxed{a_r = a_{14-r}}$$

$2 \times 7 \hookrightarrow n$

$$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = \frac{3^7 - a_7}{2}$$

$$a_0 + a_1 + a_2 + \ldots + a_6 + a_7 + a_8 + \ldots + a_{13} + a_{14} = 3^7$$

$$2\{a_0 + a_1 + a_2 + \ldots + a_6\} + a_7 = 3^7$$

Ques: $\left(1+x+x^2\right)^7 = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots\ldots$

$x \to (-x)$ $\left(1-x+x^2\right)^7 = a_0 - a_1x^1 + a_2x^2 - a_3x^3 + \ldots\ldots$

$\# \; a_0^2 - a_1^2 + a_2^2 - a_3^2 + a_4^2 \ldots\ldots =$ coeff of x^{14} in $\left(1+x+x^2\right)^7\left(1-x+x^2\right)^7$

$a_0a_0 - a_1a_1 + a_2a_2 - a_3a_3 + \ldots\ldots$

$a_0\,a_{14} - a_1\,a_{13} + a_2\,a_{12} - a_3\,a_{11} + \ldots\ldots$

coeff of x^{14} in $\left(1+x^2+x^4\right)^7 = a_7$

$\left(1+x+x^2\right)^7$

$(a_{14})x^{14}$
$(a_{13})x^{13}$
$(a_{12})x^{12}$
$(a_{11})x^{11}$

$\left(1-x+x^2\right)^7$

$(a_0)x^0$
$(-a_1)x^1$
$(a_2)x^2$
$(-a_3)x^3$

Ques: $\left(1+x+x^2\right)^7 = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots\ldots$

$x \to x^2$

$\left(1+x^2+x^4\right)^7 = a_0 + a_1x^2 + a_2x^4 + a_3x^6 +$

$a_4x^8 + a_5x^{10} + a_6x^{12} + \boxed{a_7}x^{14}$

$+ a_8x^{16} + \ldots + a_{14}x^{28}$

coeff of x^{14} in $\left(1+x+x^2\right)^7\left(1-x+x^2\right)^7$

coeff of x^{14} in $\left(1+x^2+x^4\right)^7$

OR

coeff of x^7 in $\left(1+x+x^2\right)^7 = a_7$.

SERIES

Type-05: *Replacement*

$ax^2 + bx + a \Rightarrow$ $\boxed{x \to \frac{1}{x}}$

$ax^2 + bx + c \Rightarrow$ $\boxed{x \to \frac{c}{ax}}$

Hint

Coeff. from starting & end are involved!

- Reciprocal eqn.

$$\boxed{ax^4 + bx^3 + cx^2 + bx + a}$$

$x \to \frac{1}{x}$

$2x^2 + 5x + 7$

$x \to \frac{1}{x}$

$\frac{2}{x^2} + \frac{5}{x} + 7$

$\frac{2 + 5x + 7x^2}{x^2}$

$x \to \frac{\text{Product of Root}}{x}$

$x \to \frac{7}{2x}$.

$2 \times \left(\frac{7}{2x}\right)^2 + 5 \times \frac{7}{2x} + 7$.

$2 \times \frac{49}{4x^2} + 5 \times \frac{7}{2x} + 7 = \frac{2 \times 49 + 2 \times 5 \times 7x + 4 \times 7x^2}{4x^2}$

$= \frac{2 \times 7\left(7 + 5x + 2x^2\right)}{4x^2}$

$\frac{}{2}$

BUMPER QUESTION

If n is a positive integer and $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$

Then show that :

(i) $a_r = a_{2n-r}$; $0 \le r \le 2n$

(ii) $a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$

(iii) $a_0 - a_1 + a_2 - a_3 + ... + a_{2n} = 1$

(iv) $a_0 + a_1 + a_2 + ... + a_{n-1} = \frac{1}{2}(3^n - a_n)$

(v) $a_0 + a_2 + a_4 + ... + a_{2n} = \frac{1}{2}(3^n + 1)$

(vi) [IIT-JEE 1994] $a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2 = a_n$

→ Subjective !

QUESTION [JEE Mains-2020]

Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _____.

$a_{13} = \frac{2^7 \cdot a_7}{2^{13-3}}$

$8 = 2^3 = \frac{a_7}{a_{13}}$

8

$a_0 = \frac{2^{20} a_{20}}{2^{10}}$

$a_{20} = \frac{2^0 \cdot a_0}{2^{10}}$

$a_{19} = \frac{2^1 \cdot a_1}{2^{10}}$

$(2x^2+3x+4)^{10} = a_0 + a_1x^1 + a_2x^2 + \ldots a_{19}x^{19} + a_{20}x^{20}$

$x \to \frac{2}{x}$

$\frac{2^{10}(4+3x+2x^2)^{10}}{x^{20}} = a_0 + a_1\frac{2}{x} + a_2\cdot\frac{2^2}{x^2} + \ldots\ldots + a_{20}\frac{2^{20}}{x^{20}}$

$\frac{2^{10}(4+3x+2x^2)^{10}}{x^{20}} = \frac{a_0x^{20} + 2a_1x^{19} + 2^2\cdot a_2x^{18} + \ldots\ldots + a_{20}2^{20}}{x^{20}}$

Same !!

$(4+3x+2x^2)^{10} = \frac{a_0}{2^{10}}x^{20} + \frac{2a_1}{2^{10}}x^{19} + \frac{2^2\cdot a_2}{2^{10}}x^{18} + \ldots + \frac{2^{20}a_{20}}{2^{10}}$

$a_{13} = \frac{2^4 \cdot a_7}{2^{4+3}}$

$8 = 2^3 = \frac{a_7}{a_{13}}$ (8) Ans.

$a_0 = \frac{2^{20} a_{20}}{2^{10}}$

$a_{20} = \frac{2^0 \cdot a_0}{2^{10}}$

$a_{19} = \frac{2^1 \cdot a_1}{2^{10}}$

$$(2x^2+3x+4)^{10} = a_0 + a_1x^1 + a_2x^2 + \ldots a_{19}x^{19} + a_{20}x^{20}$$

$$a_r = \frac{2^{20-r}}{2^{10}} \cdot a_{20-r}$$

$$a_7 = \frac{2^{13}}{2^{10}} a_{13}$$

$$\frac{a_7}{a_{13}} = 2^3 = (8)$$

$$(4+3x+2x^2)^{10} = \frac{a_0}{2^{10}}x^{20} + \frac{2a_1}{2^{10}}x^{19} + \frac{2^2 \cdot a_2}{2^{10}}x^{18} + \ldots + \frac{2^{20} a_{20}}{2^{10}}$$

IMPORTANT NOTE

Dhyaan se Dekho

$$\sum_{j=0}^{3}\sum_{i=0}^{3}(i^2+j^2)=$$

Symm. func^n of i & j.

$2i^2+j^2$ ✗

${}^nC_i\,{}^nC_j$ ✓

$i^nC_j + j^nC_i$ ✓

(0^2+0^2)	(1^2+0^2)	(2^2+0^2)	(3^2+0^2)
(0^2+1^2)	(1^2+1^2)	(2^2+1^2)	(3^2+1^2)
(0^2+2^2)	(1^2+2^2)	(2^2+2^2)	(3^2+2^2)
(0^2+3^2)	(1^2+3^2)	(2^2+3^2)	(3^2+3^2)

$i>j$

$i=j$

$i<j$

$i=j$

$$\sum_{j=0}^{3}\sum_{i=0}^{3}(i^2+j^2)$$

$j \to i$

$$\sum_{i=0}^{3}(2i^2)$$

$$2\left(\sum_{i=0}^{3} i^2\right)$$

DOUBLE SIGMA PROBLEMS

Important Formula : # If $f(i, j) \rightarrow$ symmetric function of i & j.

$$\sum_{j=0}^{n}\sum_{i=0}^{n} f(i,j) = \sum_{i<j}\sum f(i,j) + \sum_{j<i}\sum f(i,j) + \sum_{i=j}\sum f(i,j)$$

Total = $(i<j) + (i>j) + (i=j)$

$(i<j)$ and $(i>j)$ are equal

$$\frac{\text{Total} - (i=j)}{2} = (i<j) \text{ or } i>j$$

$(i=j) \rightarrow \sum_{i=0}^{n} f(i,i)$

BUMPER QUESTION

Find all of the following:

$$P = \sum_{j=0}^{n}\sum_{i=0}^{n}({}^nC_i + {}^nC_j) = \sum_{j=0}^{n}\left(\sum_{i=0}^{n}{}^nC_i + \sum_{i=0}^{n}{}^nC_j\right) = \sum_{j=0}^{n}\left(2^n + (n+1)\,{}^nC_j\right)$$

$$= 2^n\cdot(n+1) + (n+1)\cdot 2^{n+1}$$

$$P = 2^{n+1}\cdot(n+1)$$

$$Q = \sum_{j=0}^{n}\sum_{i=0}^{n}({}^nC_i + {}^nC_j) \text{ Where } i = j$$

OR $\sum_{i=j=0}^{n}{}^nC_i + {}^nC_j = \sum_{i=0}^{n}2\,{}^nC_i = 2\sum_{i=0}^{n}{}^nC_i = 2\times 2^n = 2^{n+1}$

$$R = \sum_{0\le i< j\le n}\sum({}^nC_i + {}^nC_j)$$

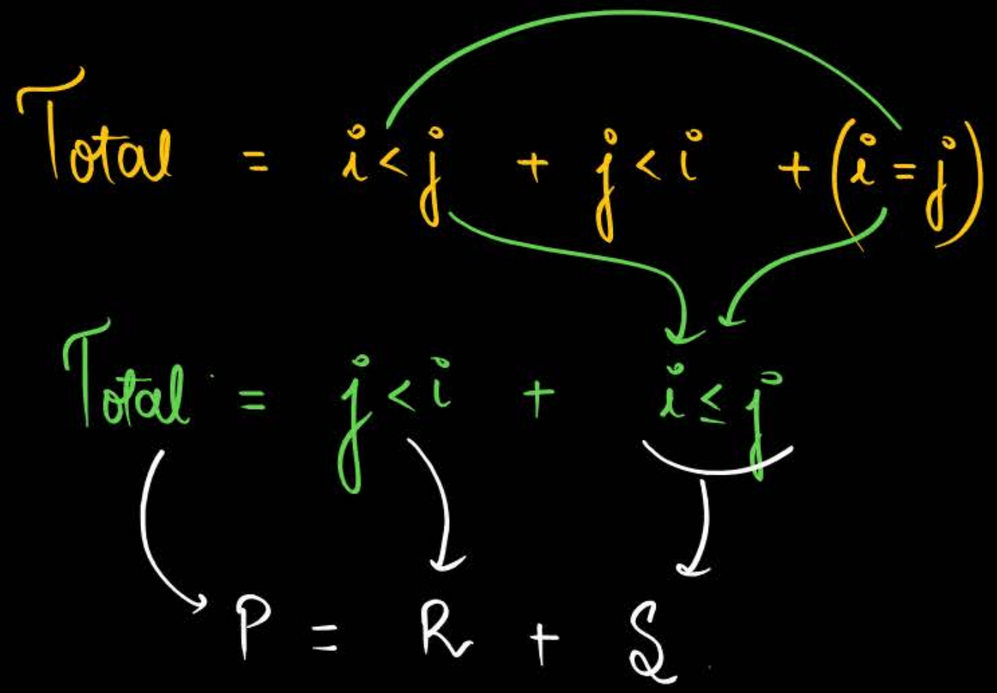
$$R = \frac{P-Q}{2} = \frac{2^{n+1}(n+1) - 2^{n+1}}{2} = \frac{2^{n+1}(n+1-1)}{2} = n\cdot 2^n$$

$$S = \sum_{0\le i\le j\le n}\sum({}^nC_i + {}^nC_j)$$

$i \le j$

$S = Q + R$

$= P - R$


Total = i<j + j<i + (i=j)
Total = j<i + i≤j
P = R + Q
P - R = Q

QUESTION [JEE Mains-2022]

$\sum_{\substack{i,j=0 \\ i\neq j}}^{n} {}^{n}C_i \, {}^{n}C_j$ is equal to

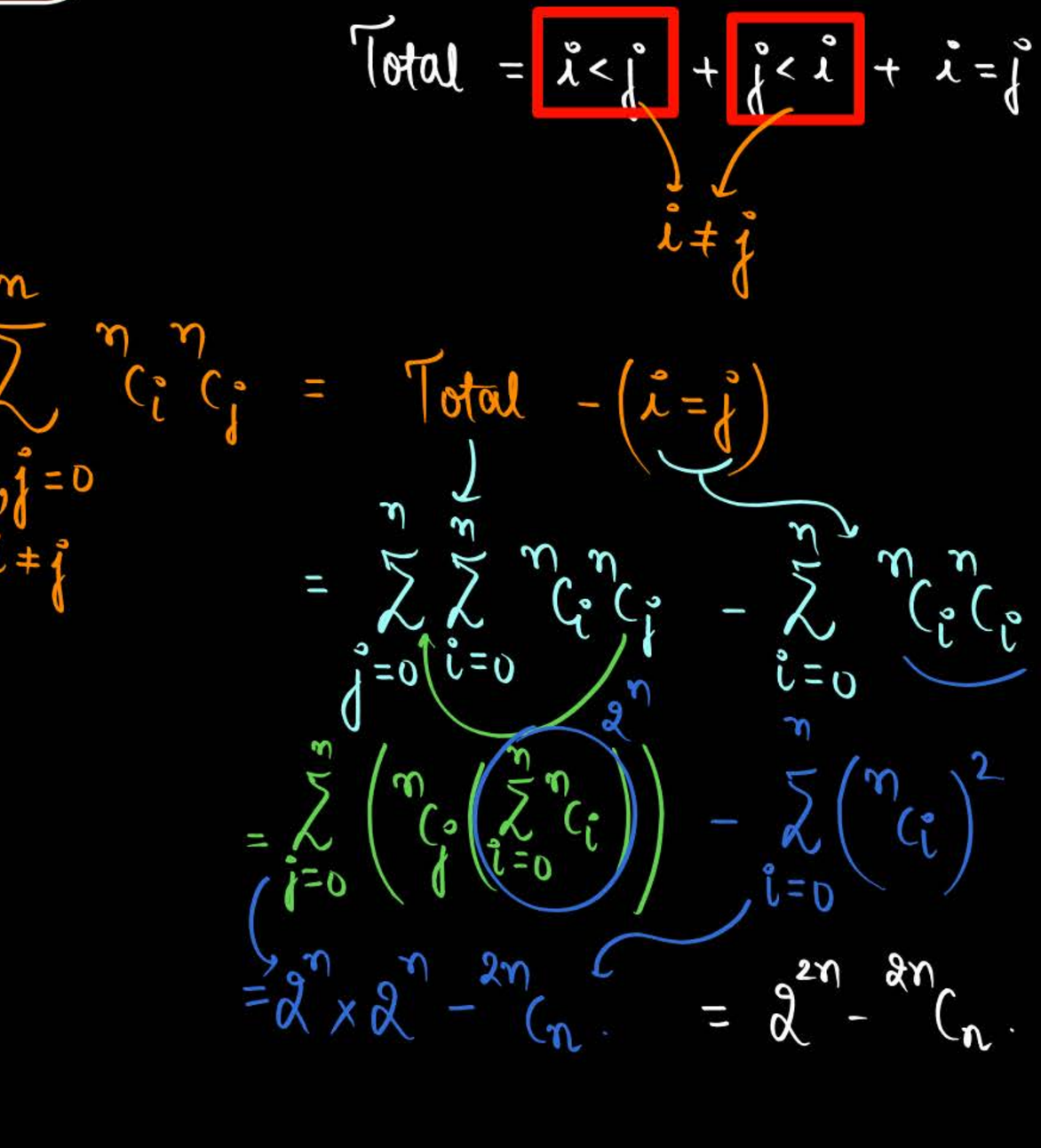
(A) $2^{2n} - {}^{2n}C_n$

(B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^n - \frac{1}{2}{}^{2n}C_n$

(D) $2^{n-1} + {}^{2n-1}C_n$

Ans: A


Total = i<j + j<i + i=j
i≠j
$\sum_{i,j=0, i\neq j}^{n} {}^{n}C_i \, {}^{n}C_j$ = Total − (i=j)
$= \sum_{j=0}^{n}\sum_{i=0}^{n} {}^{n}C_i \, {}^{n}C_j - \sum_{i=0}^{n} {}^{n}C_i \, {}^{n}C_i$
$= \sum_{i=0}^{n} \left({}^{n}C_i \left(\sum_{i=0}^{n} {}^{n}C_i \right) \right) - \sum_{i=0}^{n} \left({}^{n}C_i \right)^2$
$= 2^n \times 2^n - {}^{2n}C_n$
$= 2^{2n} - {}^{2n}C_n$

MULTINOMIAL THEOREM

$$(x+y)^n = \sum \frac{n!}{\alpha!\,\beta!}\, x^\alpha y^\beta$$

$\#\ \alpha+\beta=n$

$$(x+y+z)^n = \sum_{\alpha+\beta+\gamma=n} \frac{n!}{\alpha!\,\beta!\,\gamma!}\, x^\alpha y^\beta z^\gamma$$

$\alpha+\beta+\gamma=n$ → (1)

★ Total Terms $= {}^{n+3-1}C_{3-1} = {}^{n+2}C_2$.

$$(x+y+z+t)^n = \sum \frac{n!}{\alpha!\,\beta!\,\gamma!\,\delta!}\, x^\alpha y^\beta z^\gamma t^\delta$$

★ Total Terms $= {}^{n+4-1}C_{4-1} = {}^{n+3}C_3$.

$\#\ \alpha+\beta+\gamma+\delta=n$

$\left(\underbrace{x+y+z}_{\text{beggar's}}\right)^{7}$ — coins.

$$7+3-1\,C_{3-1} = {}^{9}C_{2}.$$

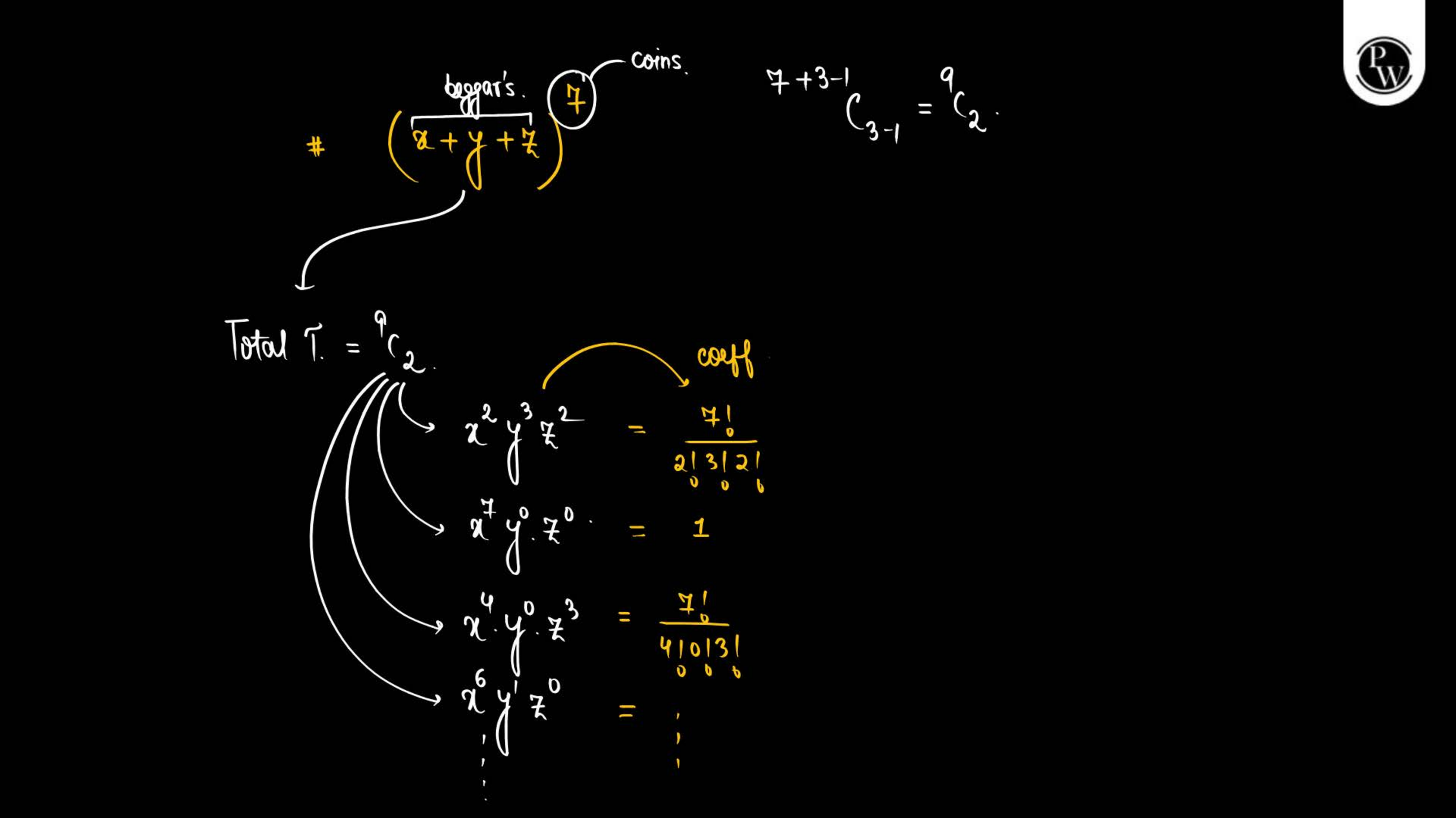
$$(x+y+z)(x+y+z)(x+y+z)(x+y+z)(x+y+z)(x+y+z)(x+y+z)$$

① $x^2y^3z^2$ = $y\ x\ x\ y\ y\ z\ z$

① $x^2y^3z^2$ = $x\ y\ x\ y\ y\ z\ z$

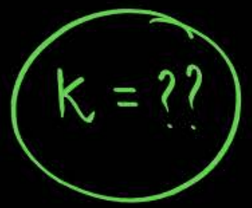
Coeff of $x^2y^3z^2 = \dfrac{7!}{2!3!2!}$

$\#\ x^3y^3z^4 = 0$


coins.
beggar's.
7
x + y + z
#
7 + 3 - 1 C 3 - 1 = 9 C 2.
Total T. = 9 C 2.
coeff
x² y³ z² = 7! / 2! 3! 2!
x⁷ y⁰ z⁰ = 1
x⁴ y⁰ z³ = 7! / 4! 0! 3!
x⁶ y¹ z⁰ =

QUESTION [JEE Mains-2021]

If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K.2^{16}$, then K is equal to_____.



$$\frac{10!}{\alpha!\beta!\gamma!}\, a^{\alpha}\,(2b)^{\beta}\cdot(4ab)^{\gamma} = \frac{10!}{\alpha!\beta!\gamma!}\, 2^{\beta}\cdot 4^{\gamma}\cdot a^{\alpha+\gamma}\cdot b^{\beta+\gamma}$$

(a^7: $\alpha+\gamma=7$; b^8: $\beta+\gamma=8$)

$$\alpha+\beta+\gamma = 10,\quad \alpha+\gamma = 7,\quad \beta+\gamma = 8$$

$$\alpha = 2,\ \beta = 3,\ \gamma = 5$$

$$\left(\frac{10!}{2!3!5!}\, 2^{3}\cdot 4^{5}\right) a^7 b^8$$

QUESTION

The coefficient of x^4 in $(3x^2 - x - 2)^{10}$ is _____.

n.o.t. $= {}^{10+3-1}C_{3-1} = {}^{12}C_2$ ✗

Sunita.

$(3x+2)^{10}(x-1)^{10}$

$(\ )x^4$	$(\ )x^0$
$(\ )x^3$	$(\ )x^1$
$(\ )x^2$	$(\ )x^2$
$(\ )x^1$	$(\ )x^3$
$(\ )x^0$	$(\ )x^4$

✓

factorisable.

Sush.

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}(3x^2)^{\alpha}(-x)^{\beta}(-2)^{\gamma}$$

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}3^{\alpha}(-1)^{\beta+\gamma}2^{\gamma}x^{2\alpha+\beta}$$

$2\alpha+\beta = 4$, $\alpha+\beta+\gamma = 10$

	α	β	γ
I	0	4	6
II	1	2	7
III	2	0	8

$$\text{Req} = \left(\frac{10!}{0!\,4!\,6!}3^0(-1)^{10}2^6 + \frac{10!}{1!\,2!\,7!}3^1(-1)^9 2^7 + \frac{10!}{2!\,0!\,8!}3^2(-1)^8 2^8\right)$$

$$\begin{matrix} 2 & 2 & 3 \\ 0 & 6 & 1 \\ 1 & 4 & 2 \end{matrix}$$

$$\left(x^2 + x + 1\right)^7 \longrightarrow \text{n.o.t.} = 14 + 1 = 15 \checkmark$$

$$7+2 C_2 = {}^{9}C_2 = \frac{9 \times 8}{2} = 36 \times$$

$$(x^2)^1 \cdot (x)^4 \cdot (1)^2 \qquad (x^2)^0 (x)^6 (1)^1 \quad (x^2)^2 \; x^2 (1)^3$$

$$\Downarrow$$

$$x^6 + x^6 + x^6$$

QUESTION [JEE Mains-2023]

The coefficient of x^7 in $(1-x+2x^3)^{10}$ is

$\beta + 3\gamma = 7$ & $\alpha+\beta+\gamma = 10$

Ans: 960

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}(1)^{\alpha}(-x)^{\beta}(2x^3)^{\gamma}$$

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}(1)^{\alpha}(-1)^{\beta}2^{\gamma}\,x^{\beta+3\gamma}$$

$\beta+3\gamma = 7$

α	β	γ
7	1	2
5	4	1
3	7	0

DIBY!!

QUESTION [JEE Mains-2023]

The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is

Ans: 1080

$$\frac{5!}{\alpha!\,\beta!\,\gamma!}(2x)^{\alpha}\left(\frac{1}{x^7}\right)^{\beta}(3x^2)^{\gamma}$$

$$\frac{5!}{\alpha!\,\beta!\,\gamma!}\,2^{\alpha}\,3^{\gamma}\,x^{\alpha-7\beta+2\gamma}$$

$\alpha-7\beta+2\gamma = 0$

$\left(\frac{5!}{1!\,1!\,3!}\,2^1\cdot 3^3\right)x^0$

$\alpha-7\beta+2\gamma=0$ OR $\alpha+2\gamma=7\beta$; $\alpha+\beta+\gamma=5$

$7\beta-2\gamma+\beta+\gamma=5$

$8\beta-\gamma=5$

	α	β	γ
*	1	1	3

QUESTION

Find sum of rational terms in $(2^{1/2} + 3^{1/3} + 5^{1/7})^{10}$.

α	β	γ
0	3	7
4	6	0
10	0	0

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}\left(2^{\frac{1}{2}}\right)^{\alpha}\left(3^{\frac{1}{3}}\right)^{\beta}\left(5^{\frac{1}{7}}\right)^{\gamma}$$

DIBY!

$\alpha+\beta+\gamma=10 \cap \frac{\alpha}{2}=I_1 \cap \frac{\beta}{3}=I_2 \cap \frac{\gamma}{7}=I_3$

$\beta = 0, 3, 6, 9, 12 \ldots$ (9 ✗, 12 ✗)

$\gamma = 0, 7, 14 \ldots$ (7 ✓, 14 ✗)

ACIDIC QUESTION

Find : $\sum_{r=0}^{12}(-1)^r \cdot a_r {}^{12}C_r$, where $\left(1+x+x^2\right)^{12} = \sum_{r=0}^{24} a_r \cdot x^r$

Hw

QUESTION [JEE Mains-2023]

Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$, $a, b, c \in N$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to

(A) 15

(B) 8

(C) 6

(D) 12

DO IT BY YOURSELF (DIBY)

DIBY-64

Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$.

$1.1\,C_1 + 3.2\,C_2 + 5.3\,C_3 + \ldots\ldots$ up to 10 terms $= \frac{\alpha \times 2^{11}}{2^{\beta}-1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots \text{upto 10 terms}\right)$, then the value of $\alpha + \beta$ is equal to

[JEE Mains-2022]

[Ans. : 286]

DIBY-65

If $\frac{1}{n+1}\,{}^nC_n + \frac{1}{n}\,{}^nC_{n-1} + \ldots + \frac{1}{2}\,{}^nC_1 + {}^nC_0 = \frac{1023}{10}$, then n is equal to [JEE Mains-2023]

(A) 9 (B) 8 (C) 7 (D) 6

[Ans. : (A)]

DIBY-66

If for some m, n; ${}^{6}C_{m} + 2({}^{6}C_{m+1}) + {}^{6}C_{m+2} > {}^{8}C_{3}$ and ${}^{n-1}P_{3} : {}^{n}P_{4} = 1 : 8$, then ${}^{n}P_{m+1} + {}^{n+1}C_{m}$ is equal to [JEE Mains-2024]

(A) 380 (B) 376 (C) 384 (D) 372

[Ans. : (D)]

DIBY-67

$30_{C_0}\ 30_{C_{10}}$

The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} \ldots + \binom{30}{20}\binom{30}{30}$ is where $\binom{n}{r} = {}^{n}C_{r}$ [IIT-JEE-2005]

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$ (C) $\binom{60}{30}$ (D) $\binom{31}{10}$

[Ans. : (A)]

DIBY-68

If $a_n = \sum_{r=0}^{n} \frac{1}{{}^{n}C_{r}}$, then value of $\sum_{r=0}^{n} \frac{r}{{}^{n}C_{r}}$ = [IIT-JEE]

(A) $2a_n$ (B) na_n (C) $\frac{2}{a_n}$ (D) $\frac{na_n}{2}$

[Ans. : (D)]

DIBY-69

Prove that $^{100}C_0\,{}^{100}C_2 + {}^{100}C_2\,{}^{100}C_4 + {}^{100}C_4\,{}^{100}C_6 + \cdots + {}^{100}C_{98}\,{}^{100}C_{100} = \frac{1}{2}\left[{}^{200}C_{98} - {}^{100}C_{49}\right]$.

DIBY-70

If $1\,{}^{100}C_6 + 4\,{}^{100}C_7 + 6\,{}^{100}C_8 + 4\,{}^{100}C_9 + {}^{100}C_{10}$ has the value of ${}^{x}C_y$, then the value of $(x + y)$ can be

(A) 114 (B) 115 (C) 198 (D) 199

[Ans. : (A), (C)]

DIBY-71

$\left(\frac{{}^{n}C_0 + {}^{n}C_1}{{}^{n}C_0}\right)\left(\frac{{}^{n}C_1 + {}^{n}C_2}{{}^{n}C_1}\right)\ldots\left(\frac{{}^{n}C_{n-1} + {}^{n}C_n}{{}^{n}C_{n-1}}\right) = \frac{(15)^{14}}{14!}$, find value of n.

[Ans. : 14]

DIBY-72

$\sum_{r=1}^{n} \sum_{p=0}^{r-1} {}^{n}C_r \cdot {}^{r}C_p \cdot 2^p$

$\sum_{r=1}^{n} \left({}^{n}C_r \left(\sum_{p=0}^{r-1} {}^{r}C_p \, 2^p \right) \right)$

[Ans. : $4^n - 3^n$]

DIBY-73

Prove that $C_0 - 2^2 \cdot C_1 + 3^2 \cdot C_2 - \cdots + (-1)^n (n+1)^2 \, C_n = 0, n > 2$, where $C_r = {}^{n}C_r$

[JEE 1989, 5M]

DIBY-74 *** Challenger!

If the value of ${}^{55}C_5 - {}^{5}C_1 \cdot {}^{44}C_5 + {}^{5}C_2 \cdot {}^{33}C_5 - {}^{5}C_3 \cdot {}^{22}C_5 + {}^{5}C_4 \cdot {}^{11}C_5$ is a^b where a and b are both prime numbers, then find $(a - b) =$

[Ans. : 6]

DIBY-75

Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$

Statement-1 : $S_3 = 55 \times 2^9$

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$ [AIEEE-2010]

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

[Ans. : (C)]

DIBY-76

Prove that $\frac{1^2}{3}\,{}^nC_1 + \frac{1^2+2^2}{5}\,{}^nC_2 + \frac{1^2+2^2+3^2}{7}\,{}^nC_3 + \cdots + \frac{1^2+2^2+\cdots+n^2}{2n+1}\,{}^nC_n = \frac{n(n+3)}{6}\,2^{n-2}$

DIBY-77

If $C_r = {}^nC_r$, then prove that $(C_0 + C_1)(C_1 + C_2)\cdots(C_{n-1} + C_n) = (C_1C_2\cdots C_{n-1}C_n)(n+1)^n/n!$.

DIBY-78

Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$.

[Ans. : (60)]

DIBY-79

The coefficient of x^4 in the expansion of $(1+x+x^2)^{10}$ is [JEE Mains-2020]

[Ans. : 615]

DIBY-80

The sum of all rational terms in the expansion of $(1+2^{1/3}+3^{1/2})^6$ is equal to ___

[JEE Mains-2025]

[Ans. : 612]

DIBY-81

If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k.\ell$, where ℓ is an odd integer, then the value of k is equal to: [JEE Mains-2022]

(A) 6 (B) 7 (C) 8 (D) 9

[Ans. : (D)]

DIBY-82

Prove that : $\sum_{s=1}^{n} \sum_{r=0}^{s} {}^{n}c_s \cdot {}^{s}c_r = 3^n - 1$

DIBY-83

Find the value of $\sum_{0 \le i < j \le n} \sum ({}^{n}C_i + {}^{n}C_j)$

[Ans. : $n2^n$]

DIBY-84

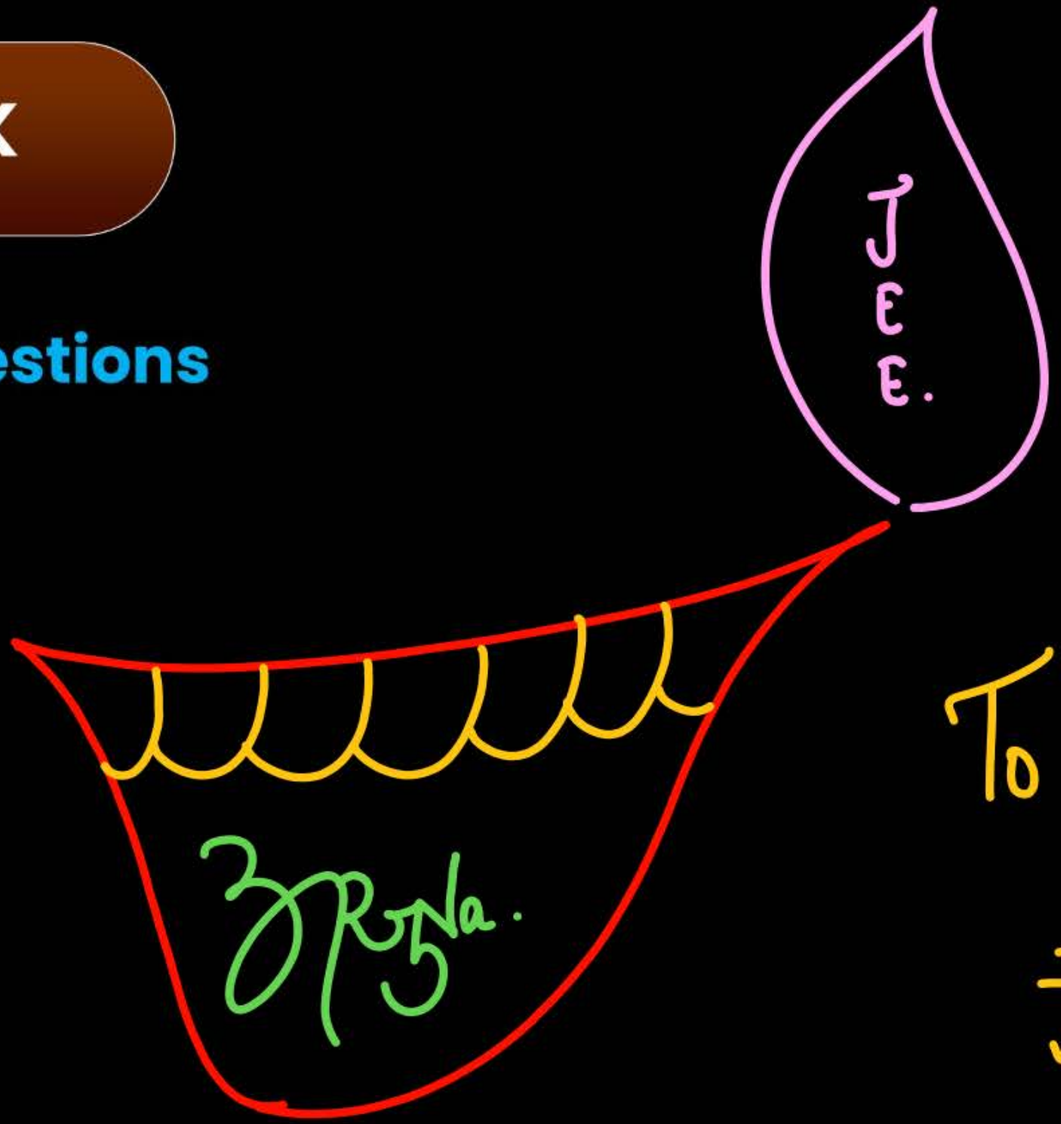
Find the value of $\sum_{0 \le i < j \le n} (i + j)({}^{n}C_i + {}^{n}C_j)$

[Ans. : $n^2 2^n$]

HOMEWORK

Solve DIBY & HW Questions

DPP



To you & your family.

Happy Diwali!

It is never about end result
It's always about the Journey

THANK You

#future IITians